Quality Improvement of Surface Meshes Using Local Parameterization

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Surface meshes play an important role in mesh generation and mesh based analysis applications. The success of mesh generation algorithms and the accuracy of numerical analyses often depend on the quality of the surface mesh. Therefore, optimization of surface mesh quality is a very important problem.

In this research, a procedure has been developed to optimize polygonal surface meshes by repositioning nodes in a series of local parametric spaces originating from a barycentric mapping of the triangular facets of the original mesh faces. The repositioning of nodes is directed by a two-stage numerical optimization procedure [2] that is designed to improve the geometric shape of mesh faces while keeping the modified mesh as close as possible to the base mesh. Repositioning nodes in the parametric space of the surface guarantees that the nodes will remain on the surface when mapped back to real space.

The first stage of the procedure is a local optimization in which the optimal (*reference*) position of each mesh vertex is calculated with respect to the fixed positions of its neighbors. The local objective function is based on Jacobian matrix condition numbers of mesh faces connected to the particular vertex [2, 1]. The reference positions of vertices are used to calculate two *reference edge vectors* for each edge in the mesh; each reference edge vector goes from the reference position of one vertex of the edge to the original position of the other. The reference edge vectors are then used to compute *Reference Jacobian Matrices* in the same way that Jacobian matrices were defined for the original mesh. Finally, the reference Jacobian matrices are used to compute an objective function for a global optimization in the second stage of the procedure. The definition of this objective function directs the the optimization to find a configuration for all the mesh edges such that a compromise is struck between the various pairs of reference edge vectors, the mesh remains valid and the element quality is improved.

The objective functions for the two optimizations are defined in terms of real coordinates of the vertices. If an optimization procedure is applied directly to these objective functions, it may indicate vertex movement off the base surface mesh. To constrain the movement of the vertices to the faces of the base mesh, the objective functions are optimized with respect to a local *barycentric* mapping of the triangular facets of the mesh faces into 2D space. All objective function evaluations are performed after transforming the parametric coordinates into real coordinates using the barycentric mapping. The gradient of the objective function with respect to the parametric coordinates is calculated using numerical differentiation. The gradient direction is used to compute a search direction, **d**, in parametric space according to the principles of the non-linear conjugate gradient method. The movement of the vertices in the parametric space is constrained by mesh validity and by the local parametric bounds of the triangular facet. If a vertex goes out of bounds of the local parametric space, the search switches to the local parametric space of the adjacent element in the mesh.

The procedure has been tested using a number of surface meshes and has proved to be very effective in improving mesh quality while minimizing changes to the surface characteristics.

References

[1] P. M. KNUPP, L. G. MARGOLIN, AND M. J. SHASHKOV. Reference Jacobian optimization-based re-zone strategies for Arbitrary Lagrangian Eulerian methods. *Journal of Computational Physics*, 176:93–128, 2002.

[2] M. J. SHASHKOV AND P. M. KNUPP. Optimization-based reference-matrix rezone strategies for Arbitrary Lagrangian-Eulerian methods on unstructured grids. In *Proceedings of the Tenth Anniversary International Meshing Roundtable*, pages 167–176, Newport Beach, CA, October 2001. Sandia National Laboratories. Sandia Report SAND 2001-2976C.

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	Initial	RJ Opt.	CN Opt.
Min. Face Angle	2.75°	6.94°	13.19°
Max. Face Angle	173.92°	151.60°	147.53°
Min. Cond. Num.	4.00	4.00	4.00
Max. Cond. Num.	98.83	34.14	19.64
Max. Hausdorff Dist. (% of problem size)		0.24%	0.48%
Max. Node Movement (% of problem size)		1.38%	2.98%

Mesh Statistics for optimization of triangular mesh of pig illustrating that the Condition Number based optimization improves the mesh more while the Reference Jacobian based Optimization keeps the mesh close to the original.
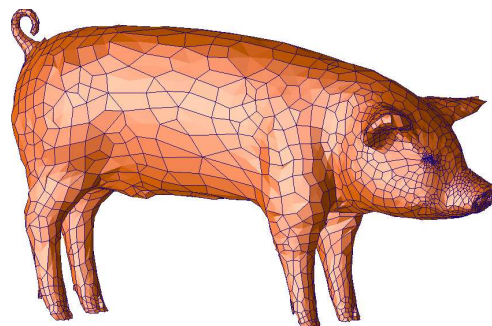
Acknowledgements
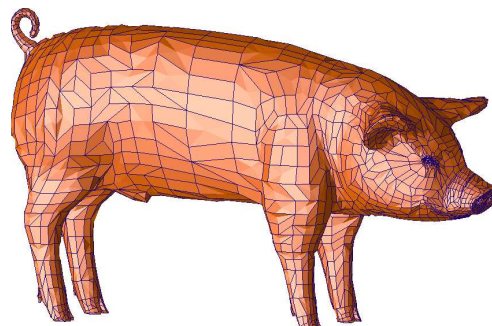
(a)

(b)
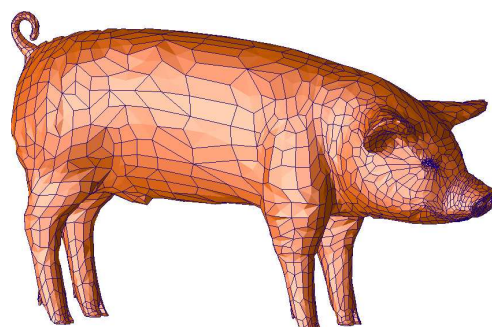
(c)

Optimization of polygonal mesh of pig showing difference between a single-stage condition number based optimization and two-stage reference Jacobian based optimization (a) Original Mesh (b) Mesh optimized with condition number objective function (c) Optimized with reference jacobian objective function.